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Frequency-Weighted System Identification and Linear Quadratic Controller Design

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Application of filters for the frequency weighting of Markov parameters (pulse response functions) is described in relation to system/observer identification. The time domain identification approach recovers a model that has a pulse response weighted according to frequency. The identified model is composed of the original system and filters. The augmented system occurs in a form that can be used directly for frequency-weighted linear quadratic controller design. Data from either single or multiple experiments can be used to recover the Markov parameters. Measured acceleration signals from a truss structure are used for system identification and the model obtained is for frequency-weighted controller design. The procedure makes the identification and controller design complementary problems.

Introduction

IDENTIFICATION of systems to match the input-output properties in certain frequency ranges is very important for controller designs. Stability robustness under system uncertainties is studied based on the fidelity of the models as a function of frequency. Usually, when identifying systems for model validation and/or load analysis, the experiment is not tailored for control design. In many cases, only one set of tests is performed and the model derived is used for multiple purposes. For control design there are stability issues that have to be met with any identified system. Issues such as good input-output fidelity in the ranges where the control system has authority are imperative. Good error bounds outside the frequency range of interest are also important. These control objectives should be considered from the beginning in any identification scheme. Identification parameters such as filtering requirements, sampling rate, bandwidth, etc., can then be properly selected.

The objectives of this paper are as follows. First, a least-squares performance criterion is developed for tailoring the Markov parameters (pulse response functions) to particular frequency ranges. This requires the introduction of filters that are combined with the actual system to form a composite system that includes the actual system and added filters. Filtering has been used for the frequency weighting of autoregressive models[1] and smoothing of transfer functions in conventional frequency domain identification techniques, but not to tailor the Markov parameters directly. Because the identified model is a composite transfer function, the second objective is to motivate the identification of such systems for the design of frequency-weighted linear quadratic (LQ) controllers.[2,3] Filters are often used in identification experiments to minimize aliasing problems or reduce the effect of the signal noise on measurement data. Therefore, one could take advantage of their use for controller design as well as identification. The third objective of this paper is to discuss an identification algorithm to recover the Markov parameters using asymptotically stable observers.[4–8] The identified Markov parameters cannot be used directly for control design; however, a state-space representation is easily obtained from realization theory.[9,10] It is during realization that the system order is determined. The proposed identification procedure is divided in two steps: first, the least-squares identification of the frequency-weighted Markov parameters, and second, realization of the state-space model using these parameters. The identification theory is presented using a single time history and then extended to multiple time histories. Experimental results are shown to illustrate the theoretical development presented in this paper. Starting from a set of input-output time histories, a frequency-weighted model of a truss structure is identified. A linear quadratic controller is then designed and implemented to verify the validity of the identified model.

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Identification Criterion for Prescribed Frequency Ranges

Consider a system that can be represented by a linear discrete time-invariant model of the form

$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k) + Du(k) \tag{1}$$

where $x(k) \in R^n$, $y(k) \in R^m$, and $u(k) \in R^r$. The identification problem seeks to find a model such that the prediction error defined by

$$\xi(k) = y(k) - \hat{y}(k) \tag{2}$$

is small. The predicted output given by $\hat{y}(k)$ is obtained using estimates of the system parameters. By assuming zero initial conditions, the predicted output can be expressed in terms of the estimated parameters and the input as

$$\hat{y}(k) = \sum_{i=0}^{k} \hat{H}_i u(k-i) \tag{3}$$

where $H_0 = D$ and $H_i = CA^{i-1}B$. Since the actual system is unknown, the parameter values $\hat{H}_i$ are estimated values. A quadratic criterion defined in terms of the prediction error is given by

$$V(l) = \sum_{i=0}^{l-1} \xi^T(i)\xi(i) \tag{4}$$

where l is the total number of sample points. Defining the parameter vector $\theta = [\hat{H}_0 \ \hat{H}_1 \cdots \hat{H}_{l-1}]$, we pose the identification problem as a minimization of the criterion in Eq. (4) with respect to the parameter vector. Equation (4) assumes all errors to be equally important. Depending on the intended use, weighting of the prediction error can be used to tailor the identified model. To address this particular problem, suppose that the prediction error is filtered. The filtered error is given by

$$\xi_f(k) = \sum_{i=0}^{k} W_{k-i}\xi(i) = y_f(k) - \sum_{\tau=0}^{i}\sum_{i=0}^{k} W_{k-i}H_{i-\tau}u(\tau)$$
$$= y_f(k) - \sum_{i=0}^{k} P_{k-i}u(i) \tag{5}$$

where W_{k-i} are the filter Markov parameters, $y_f(k)$ is the measured response filtered, and P_i are the combined system and filter parameters. For multiple-input multiple-output systems, W_{k-i} is a square diagonal matrix. If Eq. (4) is written in terms of the filtered error rather than the actual error, the performance criterion is

$$V(l) = \sum_{i=0}^{l-1} \xi_f^T(i)\xi_f(i) \tag{6}$$

If the design specification is that the model be accurate in a certain frequency range, it is more convenient to work in the frequency domain. Using the definition of the discrete Fourier transform (DFT), one can write the prediction error in terms of its spectral components as

$$\xi_f(k) = \frac{1}{l\Delta T}\sum_{i=0}^{l-1} \xi_{f,d}(i)Z(k)^i \tag{7}$$

where ΔT is the sample time, $Z(k) = e^{j2\pi k/l}$, and $\xi_{f,d}(i)$ are the spectral components. Substituting Eq. (7) into Eq. (6) yields

$$V(l) = \frac{1}{l\Delta T^2}\sum_{i=0}^{l-1} \xi_{f,d}^*(i)\xi_{f,d}(i) \tag{8}$$

where the asterisk denotes complex conjugate transpose. Equation (8) is equal to Eq. (6) but the summation now is over all of the frequency components, as opposed to sample points.

Equation (5) can be transformed to the frequency domain

$$\xi_{f,d}(k) = \Delta T \sum_{i=0}^{l-1}\sum_{l=0}^{i} W_{i-l}\xi(l)Z(k)^{-i} \tag{9}$$

Using the definitions

$$\xi_d(k) = \Delta T \sum_{i=0}^{l-1} \xi(k)Z(k)^{-i}, \qquad \bar{W}(k) = \sum_{m=0}^{\infty} W_m Z(k)^{-m} \tag{10}$$

we can write Eq. (9) as

$$\xi_{f,d}(k) = \bar{W}(k)[\xi_d(k) + R(k)] \tag{11}$$

where $R(k)$ is the transformation error in the DFT.[1] When the prediction error is periodic in the time window examined, then $R(k)$ is zero. Otherwise, the transformation error is bounded from above by a constant value $|R(k)| < \bar{R}$, where $\bar{R}$ is proportional to $1/\sqrt{l}$.[1] Substituting Eq. (11) into Eq. (8), neglecting the transformation error, yields

$$V(l) = \frac{1}{l\Delta T^2}\sum_{i=0}^{l-1} \xi_d^*(i)|\bar{W}(i)|^2\xi_d(i) \tag{12}$$

Now the question of how to select the weighting in the objective function is easily addressed. The weighting in Eq. (12) is the square magnitude of the filter transfer function evaluated at $\omega_i = i/l\Delta T$. By shaping the filter transfer function, the performance criterion is weighted accordingly. For example, if the filter is of order 1 with a break frequency of ω_b, then the prediction error is weighted such that its importance decreases at 20 dB/decade beyond ω_b.

Minimizing Eq. (12) yields the solution for the system/filter parameters P_i. The system, as seen by the input-output representation, is now a composite system. From an implementation viewpoint, it is only required to filter the sensor data prior to its use. The implications of this composite system for controller design are discussed next.

Controller Design Using the Composite System

A block diagram of the composite system is sketched in Fig. 1. Suppose that the system shown in Fig. 1 is given for controller design. In the following, the continuous-system equations are used, but the argument is equally valid for discrete systems. The output filter is denoted by $G_y(s)$ and the input filter by $G_u(s)$. The problem is to design a regulator such that the performance index

$$J = \frac{\pi}{2}\int_{-\infty}^{\infty} [y_f^*(j\omega)y_f(j\omega) + v^*(j\omega)v(j\omega)]\,d\omega \tag{13}$$

is minimized. The performance index in Eq. (13) is the standard quadratic criterion written using Parseval's theorem. Minimization of Eq. (13) is subject to the system dynamics

$$y(j\omega) = G(j\omega)u(j\omega)$$
$$= C(j\omega I - A_c)^{-1}B_c u(j\omega)$$
$$= Cx(j\omega) \tag{14}$$

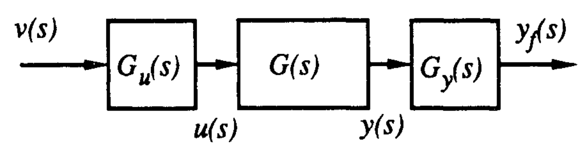

Fig. 1 Diagram of the composite system.

where $[A_c, B_c, C]$ is a realization of the transfer function if we assume that $D = 0$ and $x(j\omega)$ is the state vector. Also, Eq. (13) is subject to both input and output filter dynamics

$$u(j\omega) = G_u(j\omega)v(j\omega)$$
$$y_f(j\omega) = G_y(j\omega)y(j\omega) \tag{15}$$

For purposes of discussion, let us assume that the input filter is invertible such that

$$v(j\omega) = G_u^{-1}(j\omega)u(j\omega) \tag{16}$$

Using Eqs. (14–16) in Eq. (13) yields

$$J = \frac{\pi}{2}\int_{-\infty}^{\infty} [x^*(j\omega)C^*|G_y(j\omega)|^2 Cx(j\omega) + u^*(j\omega)|G_u^{-1}(j\omega)|^2 u(j\omega)]\,d\omega \tag{17}$$

If we start from Eq. (13), the optimal control design using the composite system corresponds to a standard linear quadratic control problem with frequency-dependence weighting defined in Eq. (17). The control problem penalizes the state and inputs as a function of frequency. The quadratic index in Eq. (17) is now replaced by Eq. (13) using the composite transfer function

$$G_c(j\omega) = G_y(j\omega)G(j\omega)G_u(j\omega) \tag{18}$$

Sample controller designs using this approach are discussed in Refs. 2 and 3. Selection of filters should be based on hardware limitations as well as stability robustness.

The foregoing discussion presents a justification for the introduction of filters from an optimal control viewpoint. In the preceding section, filters were introduced for parameter tailoring but the connection to the optimal control problem is now established. Note that the formulation just discussed is more general than what is required for parameter tailoring because the inputs could also be filtered. An experimental implementation of the frequency-weighted LQ design is presented later. The least-squares criterion for parameter estimation and the implications of the identified model for controller design having been discussed, the next section deals with the identification algorithm.

Identification of Markov Parameters

Identification from a Single Time History

The objective function for the frequency-weighted least-squares identification problem was defined in Eq. (12). An algorithm to recover the system Markov parameters using a modified system representation from that in Eq. (1) is briefly summarized here. The readers are referred to Refs. 4–8 for further details.

The identification procedure to be described treats systems in the form shown in Eq. (1). Note that any linear system can be represented in this form; therefore, the formulation is applicable to either the original system without filters or the composite system with filters. To solve for the parameters of the composite system, simply replace the input by a filtered input and the output by the corresponding filtered output.

Adding and subtracting the term $My(i)$ to the right-hand side of Eq. (1) yield

$$x(k+1) = Ax(k) + Bu(k) + My(k) - My(k)$$
$$y(k) = Cx(k) + Du(k) \tag{19a}$$

or

$$x(k+1) = \bar{A}x(k) + \bar{B}v(k)$$
$$y(k) = Cx(k) + Du(k) \tag{19b}$$

where

$$\bar{A} = A + MC$$
$$\bar{B} = [B + MD,\ -M]$$
$$v(k) = \begin{bmatrix} u(k) \\ y(k) \end{bmatrix}$$

and M is an $n \times m$ arbitrary matrix to make the matrix $\bar{A}$ a stable matrix. The form of Eq. (19b) is that of an observer if the state $x(k)$ is considered an observer state. This observer, by construction, has the same input-output properties of the system being identified. Since it is an observer equation, the Markov parameters $\bar{H}_i = C\bar{A}^{i-1}\bar{B}$ and $\bar{H}_0 = D$ to be identified are referred to as the observer Markov parameters. In matrix form, the input-output representation for l points is given by

$$\underline{y} = YV \tag{20}$$

where

$$\underline{y} = [y(0)\ y(1) \cdots y(l-1)]$$
$$Y = [\bar{H}_0\ \bar{H}_1 \cdots \bar{H}_{l-1}]$$
$$V = \begin{bmatrix} u(0) & u(1) & u(2) & \cdots & u(l-1) \\ & v(0) & v(1) & \cdots & v(l-2) \\ & & v(0) & \cdots & v(l-3) \\ & & & \ddots & \vdots \\ & & & & v(0) \end{bmatrix}$$

with the overbars denoting observer parameters. Because the matrix M is arbitrary, it can be chosen such that the observer system matrix $\bar{A}$ has certain stability properties. For example, if the matrix M places all of the observer poles at the origin, the resulting matrix $\bar{A}$ is in Jordan canonical form. If the system being identified is assumed to be of order p, then the matrix $\bar{A}$ becomes a nilpotent matrix of order p, i.e., $\bar{A}^k = 0$ for $k \geq p$. With this property, the sequence of parameters in Eq. (20) is reduced from l block matrices to $p + 1$. This significantly reduces the number of parameters since p is much smaller than l. Many other stable canonical forms can be selected for $\bar{A}$ such that different asymptotic properties are obtained.[6] For the case where $\bar{A}$ is nilpotent of order p, Eq. (20) becomes

$$\underline{y} = YV \tag{21}$$

where

$$\underline{y} = [y(0)\ y(1) \cdots y(l-1)]$$
$$Y = [\bar{H}_0\ \bar{H}_1 \cdots \bar{H}_p]$$
$$V = \begin{bmatrix} u(0) & u(1) & \cdots & u(p) & \cdots & u(l-1) \\ & v(0) & \cdots & v(p-1) & \cdots & v(l-2) \\ & & \ddots & \vdots & \cdots & \vdots \\ & & & v(0) & \cdots & v(l-p-1) \end{bmatrix}$$

A unique solution to Eq. (21) exists if the matrix V has full row rank. The row rank of the matrix also determines whether the initial assumption about the observer order is correct. If the resulting matrix V is row rank deficient, one needs to reduce the selected observer order p. The solution to Eq. (21) can be obtained in a number of different ways, as discussed in Ref. 11, but in the following discussion a pseudoinverse formulation will be used.

Note that it is not necessary to assume that the initial conditions are zero. One can write Eq. (3) or equivalently Eq.

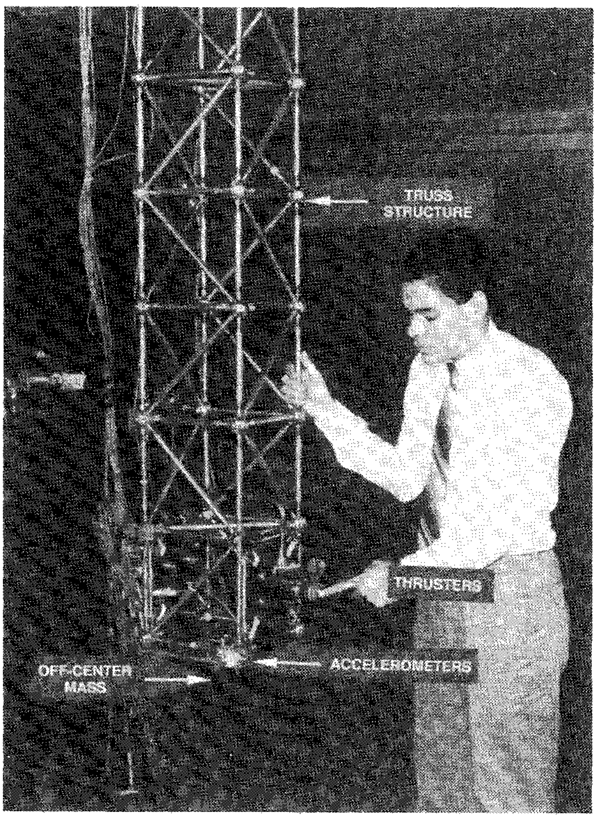

Fig. 2 Truss structure test configuration.

(20), including the term due to initial conditions. For this case, the input-output equations can be written as

$$\underline{y}_p = C\bar{A}^p\underline{x} + Y_p V_p \tag{22}$$

where

$$\underline{y}_p = [y(p)\ y(p+1) \cdots y(l-1)]$$

$$\underline{x} = [x(0)\ x(1) \cdots x(l-1)]$$

$$Y_p = [\bar{H}_0\ \bar{H}_1 \cdots \bar{H}_p]$$

$$V_p = \begin{bmatrix} u(p) & u(p+1) & \cdots & u(l-1) \\ v(p-1) & v(p) & \cdots & v(l-2) \\ v(p-2) & v(p-1) & \cdots & v(l-3) \\ \vdots & \vdots & \vdots & \vdots \\ v(0) & v(1) & v(2) & v(l-p-1) \end{bmatrix}$$

For the case where $\bar{A}^p$ is sufficiently small and all the states in x are bounded, Eq. (22) can be approximated by neglecting the first term on the right-hand side. Equation (22) is identical to Eq. (21) except that $\underline{y}$ in Eq. (21) is replaced by $\underline{y}_p$ and V by V_p. The matrices $\underline{y}_p$ and V_p are subsets of and $\underline{y}$ and V without the first p columns. The solution to Eq. (22) yields a set of observer Markov parameters. To recover the system Markov parameters from the observer Markov parameters, recall that

$$\begin{aligned} \bar{H}_i &= C\bar{A}^{i-1}\bar{B} \\ &= [C(A+MC)^{i-1}(B+MD),\ -C(A+MC)^{i-1}M] \\ &\equiv [\bar{H}_k^{(1)}, \bar{H}_k^{(2)}] \end{aligned} \tag{23}$$

$$\bar{H}_0 = H_0 = D$$

By induction, the general relationship between the Markov parameters of the actual system and the observer can be shown to be[6]

$$H_k = \bar{H}_k^{(1)} + \sum_{i=0}^{k-1} \bar{H}_{k-i}^{(2)} H_i \tag{24}$$

Knowledge of a sufficient number of the actual system Markov parameters is adequate to deduce a state-space realization of the system of interest. Physical aspects of the model such as natural frequencies, damping ratios, and mode shapes can then be found after realization of the system matrices.

Identification from Multiple Time Histories

The procedure just described deals with data analysis of a single experiment. Convergence of parameters, discussed in Ref. 8, requires sample histories that are sufficiently large. Analogous to the ergodic property of a stationary random process, one may want to replace an infinitely long time history by an infinite number of shorter time histories. The analysis of multiple experiments is derived from that of a single experiment as shown in the following.

First, consider the identification problem with a time history of $2l$ sample points long. One may write Eq. (21) as

$$[\underline{y}_1\ \underline{y}_2] = Y[V_1\ V_2] \tag{25}$$

where

$$\underline{y}_1 = [y(0)\ y(1) \cdots y(l-1)]$$

$$\underline{y}_2 = [y(l)\ y(l+1) \cdots y(2l)]$$

$$V_1 = \begin{bmatrix} u(0) & u(1) & \cdots & u(p) & \cdots & u(l-1) \\ & v(0) & \cdots & v(p-1) & \cdots & v(l-2) \\ & & \ddots & \vdots & & \vdots \\ & & & v(0) & \cdots & v(l-p-1) \end{bmatrix}$$

$$V_2 = \begin{bmatrix} u(l) & u(l+1) & u(l+2) & \cdots & u(2l) \\ v(l-1) & v(l) & v(l+1) & \cdots & v(2l-1) \\ \vdots & & \vdots & & \vdots \\ v(l-p) & v(l-p+1) & v(l-p+2) & \cdots & v(2l-p-1) \end{bmatrix}$$

The parameter vector is now computed

$$Y = (\underline{y}_1 V_1^T + \underline{y}_2 V_2^T)(V_1 V_1^T + V_2 V_2^T)^{\dagger} \tag{26}$$

where the matrix $V_1V_1^T + V_2V_2^T$ is a square matrix with $p(r+m)+r$ columns. The inverse of this matrix will be referred to as the information matrix. The rank of this matrix depends on the true system order and richness of the excitation signal. Since the true system order is unknown, to avoid

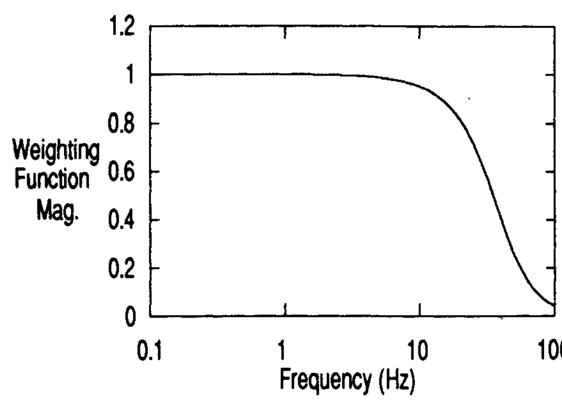

Fig. 3 Least-squares frequency-weighting function.

numerical problems, the matrix inversion operation is replaced by a pseudoinverse computation, denoted by a dagger.

Consider now the second data block to belong to a new experiment. If the system is linear time invariant, Eq. (21) holds for both experiments and the data can be appended to both sides of Eq. (21). Given the matrices $\underline{y}_1$ and V_1, where the subscript is now used to denote experiment number, the parameter estimate for the first experiment is

$$Y_1 = \underline{y}_1 V_1^T (V_1 V_1^T)^\dagger \tag{27}$$

If we use Eq. (26) as an update equation from experiment j to $j+1$,

$$Y_{j+1} = (\underline{y}_j V_j^T + \underline{y}_{j+1} V_{j+1}^T)(V_j V_j^T + V_{j+1} V_{j+1}^T)^\dagger \tag{28}$$

and adding and subtracting Y_j, Eq. (28) becomes

$$Y_{j+1} = Y_j + \hat{\epsilon}_{j+1} V_{j+1}^T (V_j V_j^T + V_{j+1} V_{j+1}^T)^\dagger \tag{29}$$

where the one experiment ahead prediction error is defined by

$$\hat{\epsilon}_{j+1} = \underline{y}_{j+1} - Y_j V_{j+1} \tag{30}$$

using the parameter vector obtained up to the jth experiment. The solution of Eq. (29) requires a matrix inversion for each additional experiment taken, but the matrix size is only a function of the assumed order of the system. The multiple experiment solution has some very interesting properties. If we assume that two experiments j and $j+1$ are given and the most recent parameter matrix is computed, the predicted output for both experiments is given by

$$[\hat{\underline{y}}_j \;\; \hat{\underline{y}}_{j+1}] = Y_{j+1}[V_j \;\; V_{j+1}] \tag{31}$$

The actual measurement can then be written in terms of the predicted output and the prediction error as

$$[\underline{y}_j \;\; \underline{y}_{j+1}] = [\hat{\underline{y}}_j \;\; \hat{\underline{y}}_{j+1}] + [\epsilon_j \;\; \epsilon_{j+1}] \tag{32}$$

By using the least-squares solution to Eq. (25), it can be shown that

$$\epsilon_j V_j^T + \epsilon_{j+1} V_{j+1}^T = 0 \tag{33}$$

if we assume that the information matrix is nonsingular. Equation (33) is a statement about the orthogonality of the prediction error with respect to all inputs and outputs for two experiments. In general, for s experiments, Eq. (33) can be extended to

$$\sum_{j=1}^{s} \epsilon_j V_j^T = 0 \tag{34}$$

Furthermore, by using Eq. (34), it can be shown that

$$\sum_{j=1}^{s} \hat{\underline{y}}_j \epsilon_j^T = 0 \tag{35}$$

The measured and estimated responses are related by

$$\sum_{j=1}^{s} \underline{y}_j \underline{y}_j^T = \sum_{j=1}^{s} \hat{\underline{y}}_j \hat{\underline{y}}_j^T + \sum_{j=1}^{s} \epsilon_j \epsilon_j^T \tag{36}$$

where again Eq. (36) is a statement of the orthogonality of predicted output and prediction error. In terms of standard least-squares analysis, the first term in Eq. (36) is the part of the responses explained by the least-squares solution, and the second term is the unexplained part. The degrees of freedom of the explained part equal the number of parameters, say, n_p, and the unexplained part has $ls - n_p$ degrees of freedom. The sample covariance of the prediction error after s experiments can be estimated by

$$\sigma^2 = \frac{1}{ls - n_p} \sum_{j=1}^{s} \epsilon_j \epsilon_j^T \tag{37}$$

This serves as a performance measure of how well the least-squares solution represents the measured data. Note that for many experiments $n_p \ll ls$, hence, the second term on the right-hand side of Eq. (36) can be neglected.

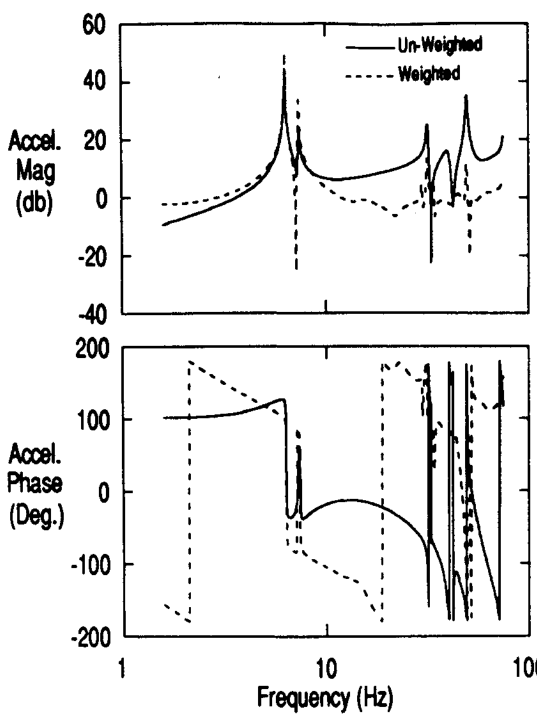

Fig. 4 Comparison of the true and weighted composite transfer function.

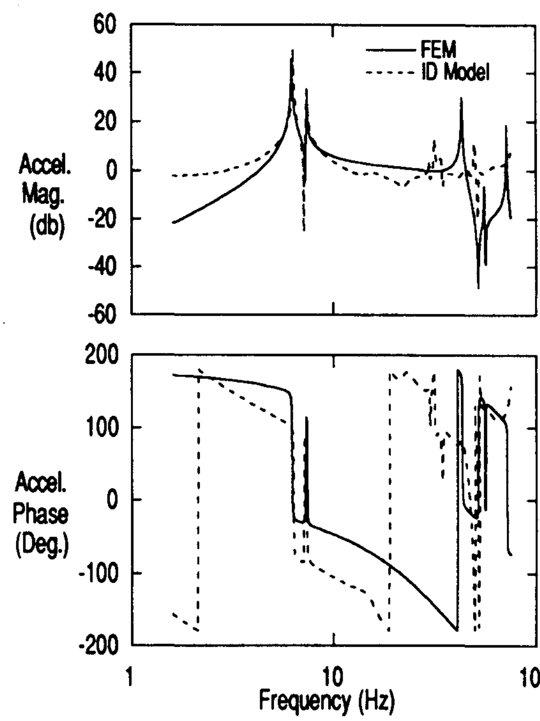

Fig. 5 Comparison of finite-element and identified models.

The identification of system Markov parameters that makes use of data from single or multiple experiments has been discussed. As mentioned earlier, the proposed identification procedure is divided into two parts: Markov parameter identification and system realization. This paper discusses the theory for Markov parameter identification. For a realization method known as the eigensystem realization algorithm (ERA), the readers are referred to Refs. 9 and 10. Both procedures are integrated in the following section on experimental results.

Table 1 Comparison of frequencies and damping values for frequency-weighted LQR; composite system includes input filter

Identified open loop		Designed closed loop		Identified closed loop	
Freq., Hz	Damp., %	Freq., Hz	Damp., %	Freq., Hz	Damp., %
6.33	0.16	6.33	2.45	6.28	2.22
7.40	0.28	7.40	3.28	7.33	2.88
29.81	0.39	29.81	0.49	29.92	0.45
31.62	0.46	31.62	0.56	31.75	0.55
34.23	0.75	34.23	0.78	34.39	0.88
49.74	0.46	49.74	0.64	49.75	0.59

Table 2 Comparison of frequencies and damping values for frequency-weighted LQR with input-output filters

Designed closed loop		Identified closed loop	
Freq., Hz	Damp., %	Freq., Hz	Damp., %
6.38	5.91	6.21	6.54
7.41	8.53	7.18	6.74
29.81	0.94	29.80	0.93
31.62	0.99	31.87	0.82
34.23	0.83	34.46	0.66
49.74	0.61	49.95	0.55

Experimental Results

To demonstrate the identification and controller design procedure, the structure shown in Fig. 2 is used. The L-shaped truss is oriented such that its longer section falls in a vertical direction extending 90 in. The shorter section, 20 in. long, is horizontal and clamped at the free end to a steel rigid plate. Two cold air thrusters acting in the same direction are placed at the tip of the truss structure. In addition, an offset weight of 30 lb is used to enhance coupling of the two principal axes and to lower the fundamental frequency of the truss. The thrusters, used for excitation and control, have a maximum thrust of 2.2 lb each. The tip accelerations are measured using two servo accelerometers placed at a corner of the square cross section.

For identification, the structure is excited using random inputs to both thrusters for 60 s. The random input signals are filtered to concentrate the energy in the low-frequency range. Three experiments with 15,000 points sampled, each at 250 Hz, are used for identification. The assumed system order is selected to be $p = 40$. The acceleration signals are filtered using a three-pole analog Bessel filter with a break frequency of 20 Hz. The resulting least-squares frequency-weighting function is shown in Fig. 3. In terms of the frequency response functions, the effect of weighting the Markov parameters is illustrated in Fig. 4. The solid and dashed curves correspond to the unweighted and weighted composite system, respectively. The weighted transfer function has a better pole-zero pattern definition in the 0–20 Hz range for identical identification parameters. A comparison of the transfer functions using a finite-element model (FEM) and the identified model is shown in Fig. 5. Correlation of the low-frequency modes is excellent; however, the upper frequency range is significantly different. This is expected for analytically derived models. The identified model used in the comparison is of order 30. Order determination for systems with noise in the data is a subtle matter. In this case, the structure has seven dominant modes corresponding to 14 states, but a model of order 30 is identified. Many of the additional states are negligible compared to the dominant modes. In fact, models have been developed with orders ranging from 16–30 and all do very well when used

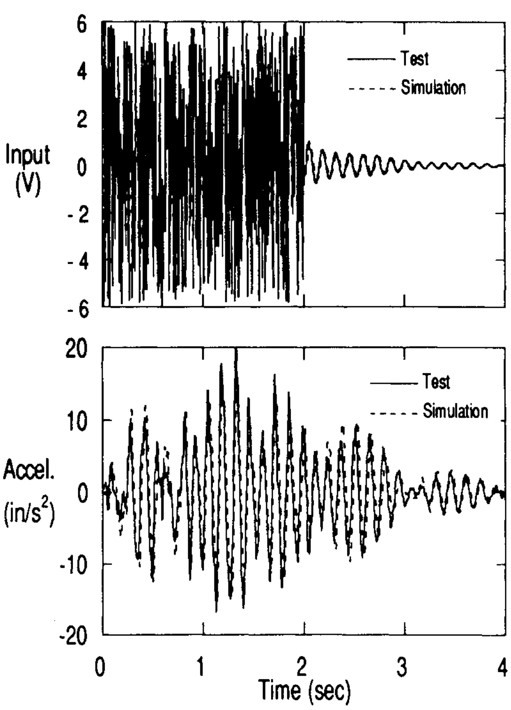

Fig. 6 Closed-loop results comparison using model with output filter.

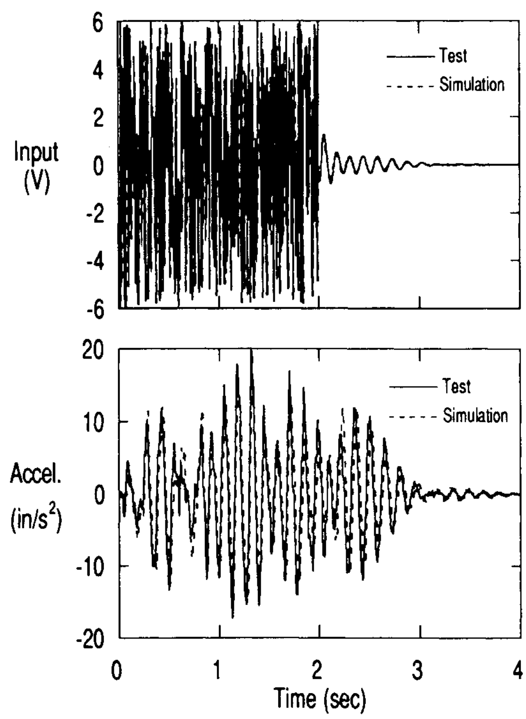

Fig. 7 Closed-loop results comparison using model with input-output filters.

to reconstruct the data. The differences are seen more in the high-frequency representation of the system transfer function. Model reduction procedures[12,13] can be applied at this stage to reduce the models further based on certain optimality conditions. The sample covariance of the prediction error, Eq. (37), after one experiment is 0.040 and reduced to 0.036 for three experiments.

Figure 6 shows a comparison of test input and acceleration signals with prediction using the 30-state identified model. The first 2 s of the test are open loop and the last 2 s closed loop. The composite system used for the design has an output filter. The dashed line represents simulation and a solid line the test. Although there are some visible differences, the reconstruction shows that the model predicts open- and closed-loop responses well. Table 1 shows the open-loop, designed closed-loop, and identified closed-loop eigenvalues for the first six structural modes. For linear quadratic regulator (LQR) design, the composite model is put in block diagonal form to penalize further the modes shown in Table 1. The diagonal elements in the state weighting matrix $\boldsymbol{Q}$ have values of 50 for the states corresponding to those in Table 1 and 10 for all other states. The input weighting matrix is $\boldsymbol{R} = \mathrm{diag}(0.1, 0.1)$. The stability of high-frequency modes is marginal and some are actually being destabilized. This is because the fidelity of the identified model decreases beyond 20 Hz.

Figure 7 shows a comparison for the case when the composite system has both input and output filters. The input filter is a three-pole Butterworth filter, whereas the output filter is a three-pole Bessel filter. The frequency cutoff is 20 Hz for both filters. The structure is excited for 2 s, followed by 2 s of closed-loop response. The designed and identified closed-loop eigenvalues are given in Table 2. In this case, the state weighting matrix $\boldsymbol{Q}$ (for the block diagonal system) has 0.01 for states corresponding to the first two modes in Table 2 and 0.001 for all others. The input weighting matrix is $\boldsymbol{R} = \mathrm{diag}(2,2)$. For modes below 20 Hz, damping augmentation is accomplished within 20% of the predicted values. Outside the 0–20 Hz range, the fidelity of the identified model degrades and therefore the design is very conservative in those ranges.

Concluding Remarks

The paper presents an approach for the identification of models with Markov parameters (pulse responses) accurate in prescribed frequency ranges. The identification procedure consists of two steps. First, a frequency-weighted least-squares solution of the Markov parameters is obtained. Second, a state-space realization of a frequency-weighted model is obtained from the identified Markov parameters. The discussion deals with the first step of the identification procedure. The second step makes use of established realization algorithms. In this paper, the frequency-weighted solution of the Markov parameters is obtained in the time domain using the concept of observer identification, which requires the identification of a parameter set that is a combination of the system and filter. Both formulations for single or multiple time histories are treated. It is shown that the resulting system, referred to as a composite system, is a basis for frequency-weighted linear quadratic controller design. The proposed methodology makes identification a complementary part of the optimal controller design problem. The method is applied to identify and control an experimental truss structure. Close agreement between test and simulated results based on the identified frequency-weighted model is demonstrated.

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